Heat transfer and melting in subglacial basaltic volcanic eruptions: implications for volcanic deposit morphology and meltwater volumes

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Abstract: Subglacial volcanic eruptions can generate large volumes of meltwater that is stored and transported beneath glaciers and released catastrophically in jökulhlaups. At typical basaltic dyke propagation speeds, the high strain rate at a dyke tip causes ice to behave as a brittle solid; dykes can overshoot a rock–ice interface to intrude through 20–30% of the thickness of the overlying ice. The very large surface area of the dyke sides causes rapid melting of ice and subsequent collapse of the dyke to form a basal rubble pile. Magma can also be intruded at the substrate–ice interface as a sill, spreading sideways more efficiently than a subaerial flow, and also producing efficient and widespread heat transfer. Both intrusion mechanisms may lead to the early abundance of meltwater sometimes observed in Icelandic subglacial eruptions. If meltwater is retained above a sill, continuous melting of adjacent and overlying ice by hot convecting meltwater occurs. At typical sill pressures under more than 300 m ice thickness, magmatic CO2 gas bubbles form c. 25 vol% of the pressurized magma. If water drains and contact with the atmosphere is established, the pressure decreases dramatically unless the overlying ice subsides rapidly into the vacated space. If it does not, further CO2 exsolution plus the onset of H2O exsolution has the potential to cause explosive fragmentation, i.e. a fire-fountain that forms at the dyke–sill connection, enhancing melting and creating another candidate pulse of meltwater. The now effectively subaerial magma body becomes thicker, narrower, and flows faster so that marginal meltwater drainage channels become available. If the ice overburden thickness is much less than c. 300 m the entire sill injection process may involve explosive magma fragmentation. Thus, there should be major differences between subglacial eruptions under local or alpine glaciers compared with those under continental-scale glaciers.

Subglacial volcanic eruptions have been studied extensively in Iceland (Björnsson 1975; Allen 1980; Gudmundsson & Björnsson 1991; Gudmundsson et al. 1997; Johannesson & Sæmundsson 1998) due to the ongoing nature of the process and the many beautifully exposed landforms and deposits. Of particular interest is the generation of large volumes of meltwater, its storage and transport below the glaciers, and the catastrophic meltwater release at glacial margins to produce jökulhlaups (Björnsson 1975, 1992). Documentation of the products and landforms resulting from these eruptions (Björnsson 1975; Allen 1980; Grönvold & Johannesson 1983; Gudmundsson et al. 1997; Johannesson & Sæmundsson 1998) and continuing study of active examples (Gudmundsson et al. 1997), together with the development of qualitative and quantitative models of the processes (Einarsson 1966; Gudmundsson et al. 1997; Höskuldsson & Sparks 1997; Hickson 2000; Smellie 2000), has led to the recognition of candidates for these processes elsewhere on Earth (Mathews 1947; Skilling 1994; Smellie & Skilling 1994; Chapman et al. 2000) and on Mars (Allen 1979; Hodges & Moore 1994; Head & Wilson 2002).

Dykes represent the propagation, both laterally and vertically, of sub-vertical magma-filled cracks from crustal or subcrustal reservoirs into the surrounding area. Dykes may propagate to the surface to cause eruptions; may propagate to the near-surface to set up stress fields, which under suitable conditions result in graben (Mastin & Pollard 1988; Rubin 1992); or may stall and cool in the crust at depths too great to produce visible indications of their presence. The latter includes the possibility that they may cease vertical propagation at some relatively shallow depth and then spread sideways to produce sills. This process is encouraged if the least principal stress ceases to be horizontal and becomes vertical. The discontinuity in density and other material properties provided by the contact between a glacier or ice-cap and the underlying

rocks may also be a trigger for such activity, and subglacial eruptions are likely to begin with the intrusion of a sill at the rock–ice boundary.

Commonly, a subaerial basaltic eruption is initially manifested as a curtain of fire along a fissure tens to hundreds of metres long which marks the surface trace of the dyke. Cooling along the narrow parts of the dyke (Wilson & Head 1988) causes localization of extrusion within a few hours to a few days, and transition to a centralized vent eruption (Head & Wilson 1987; Bruce & Huppert 1989). In submarine (Head et al. 1996) and subglacial basaltic eruptions, a classical initial curtain of fire does not generally occur because of the inhibition of gas exsolution due to the pressure of the overlying water or ice. In submarine eruptions, the suppression of gas release continues throughout the eruption, but in subglacial eruptions the situation may become much more complex. Melting of the ice overlying the initial sill may form a cavity. As long as the overlying ice does not deform too quickly, the pressure in the cavity may be less than the lithostatic load which acted on the sill during the early stages of the intrusion process, and this may lead to an increase in gas exsolution and magma vesiculation, possibly resulting in magma fragmentation and some form of explosive activity. The overlying ice cover may be completely removed, exposing magma to the pressure of the atmosphere and leading to more vigorous explosive activity.

With suitable additions, existing physical models for the ascent and eruption of magma (Wilson & Head 1981, 1983) can be applied to subglacial environments. Here we develop some simple physical principles for the intrusion of magma into a glacial cover and assess the implications for eruption behaviour and the nature of the resulting volcanic deposits and meltwater release processes. We discuss the conditions under which hyaloclastites and lava breccias form, and show how either lava flow units or sill-like bodies can form at the base of the ice, depending on the melting rate and behaviour of the ice dictated by its thickness.

**Subglacial and englacial dyke emplacement**

Mafic dykes sourced in crustal magma reservoirs are driven upward by magma buoyancy, by the presence of an excess pressure in the reservoir, or by a combination of the two. We shall show in later sections that typical mafic magmas have bulk densities smaller than those of their host rocks by $\Delta \rho = c.200 \text{ kg m}^{-3}$, so that the buoyancy pressure gradient acting on them ($g \Delta \rho$) is $c.2000 \text{ Pa m}^{-1}$. Excess pressures in crustal mafic magma reservoirs are typically $c.3 \text{ MPa}$ (Parfitt 1991) and for reservoir depths of a few kilometres these correspond to similar pressure gradients of $c.1000 \text{ Pa m}^{-1}$. The consequence is that the magma in mafic dykes with typical widths of $c.1 \text{ m}$ propagates upward at speeds of $c.1 \text{ m s}^{-1}$ (Wilson & Head 1981). The strain rates near the dyke tips implied by these speeds are $c.1 \text{ s}^{-1}$, about seven orders of magnitude larger than the strain rates at which the surrounding ice can flow plastically given the rheological models (a pseudo-plastic power-law fluid with a yield strength) proposed by Glen (1952), Nye (1953) and Paterson (1994). Thus a dyke can easily overshoot an ice–rock interface because the ice appears to the propagating crack as a brittle, low-density rock with elastic properties similar to those of the basalt substrate. We show that the amount of ice melting which takes place on the timescale of dyke emplacement may be small enough for the emplacement process to be stable, though subsequent, more extensive ice melting may lead to collapse of the dyke.

The pressure distribution in a dyke propagating through an elastic medium is dictated by several requirements that must be met simultaneously. Most fundamental is that the distribution of stress across the dyke wall (dictated by both the internal pressure distribution and the external stress distribution) must be such as to hold open the sides of the fracture into which magma is moving. There must also be a vertical pressure gradient in the magma to support the static weight of the magma, and an additional pressure gradient in the direction of magma travel to drive the motion against wall friction. To maximize the flow speed, and hence the mass and volume fluxes through a dyke of a given shape, the pressure in the propagating tip of the dyke, $P_{\text{tip}}$, must decrease to a low value. The theoretical ideal tip pressure is zero, but Rubin (1993) suggested that tip pressure would in fact be no smaller than the pressure at which the most soluble volatile species which the magma contains, commonly water, becomes saturated. The argument is that if the pressure falls slightly below the value at which the magma is saturated in this volatile, more of the volatile exsolves. The solubility function for water in basalt (Wilson & Head 1981) is:

$$n_w = K_w P^{0.7}$$

where the constant $K_w$ is $6.8 \times 10^{-8}$ if $n_w$ is expressed as a mass fraction and $P$ is the pressure in Pascals. If the magma contains 0.25 mass% water, a plausible value for a mafic magma (Gerlach 1986), $n = 0.0025$ and the saturation pressure, and hence the propagating
dyke tip pressure, is close to 3.3 MPa; we use this value in many subsequent calculations. We note, however, that reducing the assumed water content by a factor of two would imply a pressure of 1.2 MPa whereas increasing it by a factor of two would imply 9.0 MPa. We comment on the implications of this later.

When a vertically-propagating dyke comes to rest with its tip at some point below the surface, the pressure gradient due to the motion will by definition have vanished. In general, any excess pressure originally present in the magma reservoir and driving the intrusion will also have vanished, though residual pressure gradients may still be present if the magma in any part of the dyke system has a non-Newtonian rheology involving a finite yield strength (Parfitt & Wilson 1994). Figure 1 shows the configuration of such a dyke propagating from a reservoir at depth \( z \) below the rock–ice interface. A layer of ice of thickness \( y \) exists at this location and the tip of the dyke comes to rest at a depth \( x \) below the ice surface. The density \( \rho_i \) of the ice is \( 917 \text{ kg m}^{-3} \). The density \( \rho_r \) of the crustal rocks is controlled by their likely origin as a mixture of vesicular lavas and possibly poorly packed pyroclastics which have undergone various kinds of weathering and alteration: we assume a value of \( 2300 \text{ kg m}^{-3} \), close to that implied by the inversion of seismic data (Hill 1969; Zucca et al. 1982; Gudmundsson 1987; Head & Wilson 1992). To estimate the average magma density between the reservoir and the trapped tip we recall that the tip pressure is likely to be buffered by \( \text{H}_2\text{O} \) exsolution so that the only exsolved volatile phase will be \( \text{CO}_2 \) present as bubbles of gas or supercritical fluid in the magma. We assume that \( z \) is likely to be in the range 1 to 3 km, based on the depths of shallow magma reservoirs in Iceland (Björnsson et al. 1977), and that \( y \) will lie in the range 500 to at most 2000 m based on ice cap thicknesses under current (up to c. 900 m Sigmundsson & Einarsso 1992; Einarsso 1994) and glacial (1000–1500 m Einarsso & Albertsson 1988; Geirsdottir & Ericksson 1994; Bourgeois et al. 1998) conditions. The lithostatic pressure \( P_l \) at the top of the relaxed magma reservoir will then lie within the extremes of 27 and 86 MPa. The solubility \( n_c \) of \( \text{CO}_2 \) in basaltic magmas is given by (Harris 1981)

\[
\begin{align*}
 n_c &= J_c + K_c P \\
 n_c &= 3.4 \times 10^{-6} + 6 \times 10^{-12} \text{ Pa}^{-1}
\end{align*}
\]

where \( J_c \) equals \( 3.4 \times 10^{-6} \) and \( K_c \) equals \( 6 \times 10^{-12} \text{ Pa}^{-1} \) when \( n_c \) is expressed as a mass fraction. Assuming a plausible basaltic magma content \( n_t \) of this volatile, say 0.2 mass\% i.e. 0.002 mass fraction, the mass fraction exsolved at 27 MPa is 0.00183 and at 86 MPa is 0.00148. At the dyke tip, where the pressure is likely to be no less than the value during propagation \( (P_{pt} = 3.3 \text{ MPa}) \), the amount of \( \text{CO}_2 \) exsolved will be \( n_e = (n_t - n_c) = 0.00198 \). The bulk density \( \beta \) of the magma is given by

\[
\frac{1}{\beta} = \frac{n_c/\rho_c + [(1 - n_c)/\rho_m]}{(1 - n_c)}
\]

where \( \rho_c \) is the density of the \( \text{CO}_2 \) given to an adequate approximation by the ideal gas law:

\[
\rho_c = \frac{(m_c P)}{(QT_m)}.
\]

Here \( m_c \) is the molecular mass of \( \text{CO}_2 \), \( 43.99 \text{ kg kmol}^{-1} \), \( Q \) is the universal gas constant, \( 8.314 \text{ kJ kmol}^{-1} \text{ K}^{-1} \), \( T_m \) is the magma temperature, 1473 K (1200°C), and \( \rho_m \) is the density of the basaltic magmatic liquid, say 2700 kg m\(^{-3}\). Using these values, the magma bulk density varies from 1863 kg m\(^{-3}\) at 3.3 MPa to 2574 kg m\(^{-3}\) at 27 MPa to 2669 kg m\(^{-3}\) at 86 MPa. The mean bulk density, \( \beta_m \), of the magma in the dyke between the pressure in the tip, \( P_{pt} \), and at the reservoir roof, \( P_r \), is evaluated from

\[
\beta_m = (P_r - P_{pt})^{-1} \int_{P_{pt}}^{P_r} \beta dP
\]

Using equations (2) and (3), and defining the convenient constants

\[
\begin{align*}
 a &= \rho_m QT_m (n_t - J_c), \\
b &= [m_c(1 - n_t + J_c) - \rho_m QT_m K_c], \\
c &= [m_c K_c], \\
e &= \rho_m/(2K_c), \\
h &= [(b^2 - 4ac)^{1/2}], \\
f &= [a/(2K_c h)],
\end{align*}
\]
we find:

\[
\beta_m = (P_r - P_{pt})^{-1} \\
\times \left[ e \ln \left( \frac{a + bP_r + cP_t^2}{a + bP_{pt} + cP_{pt}^2} \right) \\
- f \ln \left( \frac{(2cP_r + b - h)}{(2cP_{pt} + b + h)} \times \frac{P_{pt} + b + h}{P_{pt} + b - h} \right) \right].
\] (5b)

If the roof of the magma reservoir is at the \( P_r = 27 \) MPa level, the mean density of magma in the dyke will be \( c. 2390 \) kg m\(^{-3}\) and if the reservoir roof pressure is \( 86 \) MPa the mean dyke magma density will be \( c. 2567 \) kg m\(^{-3}\). Whatever the geometry, therefore, the mean density of the magma will lie within \( c. 4\% \) of the value \( \beta = 2480 \) kg m\(^{-3}\). This value will change if the assumed magma water content is changed, because the dyke tip pressure during dyke emplacement will be buffered at a different value. Equation (1) shows that varying the water content between 0.125% and 1% causes the propagating tip pressure \( P_{pt} \) to vary between 1.2 MPa and 24 MPa. Equation (2) shows that the exsolved amount of CO\(_2\) would change by \( c. 7\% \) as a result. The change in tip pressure is thus somewhat more important than the resulting change in CO\(_2\) content of the magma. The mean magma densities corresponding to \( P_{pt} = 1.2 \) and 24 MPa are 2361 and 2564 kg m\(^{-3}\), respectively, if \( P_r = 27 \) MPa, and are 2553 and 2635 kg m\(^{-3}\), respectively, if \( P_r = 86 \) MPa, typically a 4% variation for the smaller \( P_r \) and a 1.6% variation for the larger \( P_r \).

As the tip of the dyke comes to rest, the pressure in the gas in the tip cavity will increase from the low, buffered value maintained during magma motion and will reach a final value \( P_t \).

**Fig. 2.** The pressure, \( P_t \), in the gas pocket in the tip of a dyke after it has been intruded into an ice layer (see Fig. 1) as a function of the depth, \( x \) of the tip below the ice surface and the thickness, \( y \) of the ice layer. The horizontal broken line indicates the smallest pressure, for the chosen magma volatile content (see text), likely to exist in the dyke tip while it is propagating. The inclined solid line shows the location of the ice–rock interface.
This pressure can be found by assuming that the hot rocks near the roof of the magma reservoir cannot support significant stresses (whereas the cold rocks or ice around the tip of the dyke can support stresses as large as their mechanical strengths). The balance between lithostatic \(g(\rho_z + y_\rho)\) and magma \((P_t + g\beta(z + (y - x)))\) stresses at the roof of the magma reservoir (see Fig. 1) implies, after collecting terms, that

\[
P_t = g(\beta x - (\beta - \rho_i)y + (\rho_t - \beta)z).
\]  

(6)

Using the densities adopted above, we find \((\beta - \rho_i) = 1333 \text{ kg m}^{-3}\) and \((\rho_t - \beta) = 50 \text{ kg m}^{-3}\). The relatively small value of \((\beta - \rho_i)\) means that \(P_t\) is only weakly dependent on the magma reservoir depth, \(z\) and is controlled mainly by the ice thickness, \(y\) and the depth of the dyke tip below the surface, \(x\). Figure 2 shows how \(P_t\) varies with \(x\) for \(y = 500, 1000, 1500\) and 2000 m. The horizontal line on this graph shows the water pressure of 3.3 MPa, the pressure in the dyke tip before it came to rest: any decrease in pressure below this value would lead to additional exsolution of water from the magma. This would lead to a decrease in magma density near the dyke tip but would not greatly change the mean magma density used in the calculation. The oblique line on the graph shows the boundary between dyke tips located within the ice layer (below the oblique line) and those located within the silicate rock crust (above the line). Clearly there is a wide range of conditions under which a dyke could penetrate, and stall within, the ice.

The pressure in the dyke tip in excess of the local lithostatic load of the overlying ice, \(P_e\), is equal to \((P_t - g_\rho x)\) and so using equation (6)

\[
P_t = g(\beta_0(x - y) + (\rho_t - \beta)z).
\]  

(7)

Figure 3 shows how \(P_e\) varies with \(x\) for the same set of values of \(y\) as Figure 2. Physically, \(P_e\) may be either positive or negative. The boundary between dyke tips in ice and dyke tips in

![Fig. 3. The difference in pressure, \(P_e\), between the gas in an intruded dyke tip and the external lithostatic load for the intrusion geometries corresponding to Figure 2. See text for discussion.](http://sp.lyellcollection.org/)
rock, which corresponds to setting $x$ equals $y$ in equation (7), is now a horizontal line. The line shown in Figure 3 corresponds to $z = 2$ km at $P_e = 0.98$ MPa; equation (7) shows that the corresponding values of $P_e$ for $z = 1$ km and 3 km are 0.49 and 1.47 MPa, respectively. Thus, for all cases where the dyke tip penetrates into and stalls within the ice, the excess pressure in the tip can be positive but less than about 0.5–1.5 MPa, the exact value depending on $z$. The requirement that $P_t$ be no less than c. 3.3 MPa leads to the truncation of the lines in Figure 3, and so the excess pressure $P_e$ can also become negative by up to about −10 MPa.

None of the dykes modelled above (using plausible magma densities and volatile contents) are expected to break through to the upper surface of the ice. Thus magmatic eruptions at the surfaces of glaciers and ice-caps should not be a common occurrence even when dykes do penetrate into overlying ice. However, there are some potential consequences of the fact that the pressure differential, $P_e$, between the water vapour in a dyke tip and the surrounding ice could range from c. 1.5 MPa positive to c. 10 MPa negative. Positive pressure differentials this small will probably not lead to brittle failure of the surrounding ice, being less than the likely tensile strength of the ice, but large negative pressure differentials may lead to failure in tension or shear of the ice forming the dyke walls and collapse of blocks of ice into the gas cavity at the dyke tip. This process would be encouraged by the c. 8% volume decrease which occurs when ice melts to water. Progressive collapse might occur until a pressure path to the surface was formed, in which case the excess water vapour pressure in the dyke tip would be vented to the atmosphere and the consequent unloading of the magma would lead to further magma vesiculation and the onset of explosive activity. This activity would almost certainly be phreatomagmatic because of the intimate contact between magma, water and spalled blocks of ice. It would not be long-lived, however: even complete relaxation of the pressure at the top of the magma column to atmospheric pressure would not cause magma to rise to the surface of the ice, and so the magma at the top of the column would rapidly be chilled, causing explosive activity to cease.

We can obtain an idea of the timescale for the dyke emplacement process using the typical magma rise speed, c. 1 m s$^{-1}$, quoted earlier. Figure 1 shows that dykes will penetrate a distance $(y-x)$ into the ice layer. Figure 2 shows that, as $y$ takes the values 500, 1000, 1500 and 2000 m, the value of $x$ at which $P_t$ is equal to the buffered value of c. 3.3 MPa takes the values c. 400, c. 700, c. 1000 and c. 1300 m respectively. Thus the penetration distances are $[(y-x)=]_0$ about 100, 300, 500 and 700 m respectively. At a magma rise speed of 1 m s$^{-1}$, the corresponding dyke emplacement times would range from about 100 to 700 s and in these time intervals any temperature changes caused solely by thermal conduction would penetrate a distance (d) of order $(\kappa t)^{1/2}$ where $\kappa$ is the thermal diffusivity of the ice or chilling dyke magma. Thermal diffusivities of both ice and basalt are c. $10^{-6}$ m$^2$s$^{-1}$ and so $d$ would be at most a few centimetres. Thus englacial dykes could well be emplaced in the initial phase of an eruption (Fig. 1).

Soon after their emplacement, dykes intruded into ice would provide relatively efficient ice melting because of the formation of two broad and extensive surface areas (the sides of the dyke) in contact with the ice. Anticipating calculations given below for heat loss from a sill, typical average heat transfer rates during the first 10 seconds after emplacement exceed 3 MW m$^{-2}$, and this could be a factor in the rapid initial production of meltwater reported in some Icelandic eruptions (Gudmundsson et al. 1997). Over the subsequent few tens of hours, solidification of the magma and formation of cooling cracks, together with melting of adjacent ice, would almost certainly cause the magma column to lose coherence and collapse to form a ‘dyke rubble pile’. If the dyke were c. 200 m high and c. 1 metre wide (200 m$^2$ cross-sectional area), then its eventual collapse could produce a rubble pile at least c. 15 m wide by 15 m high even with minimal bulking (or more likely c. 20 m wide by c. 10 m high if it eventually attained angle of rest slopes). The cores of eruptive structures beginning with this type of event might contain a breccia pile with morphology diagnostic of its dyke-induced origin.

We now turn our attention to the consequences of magma intruding at the ice–rock interface instead of propagating as a dyke into the ice.

**Sill intrusion at the ice–basalt substrate interface**

The conditions that determine where the tip of an initially vertically propagating dyke ceases to move upwards, and instead initiates a fracture propagating sideways to allow the intrusion of a sill, are complex. Lister (1990), in modelling the rise of mafic magmas from deep levels, has argued that lateral intrusion will be favoured at the level of magma neutral buoyancy, and this is
an attractive model for the origin of crustal magma reservoirs (e.g. Ryan 1987). However, at shallower levels in general, and especially when the tip of a dyke is nearing the shallowest level to which the stresses controlling it will allow it to penetrate, it seems inevitable that local variations in host rock properties will also play a part. We infer that dykes capable of penetrating a significant distance into overlying ice will not be excessively sensitive to the presence of the large density contrast at the rock–ice interface, whereas those which would otherwise have stalled just above the interface will initiate sill intrusion even if part of the magma rises into a dyke somewhat overshooting the interface.

When magma rises in a dyke and then intrudes as a sill, there must be a finite vertical pressure gradient in the sub-vertical feeder dyke due to the weight of the magma; if the sill is intruded horizontally, there is of course no pressure gradient in the sill due to magma weight. However, a pressure gradient required to overcome wall friction associated with magma flow must exist in both the vertical and the horizontal parts of the system. Figure 4 shows the geometry at various stages during the sill injection. The injection pressure $P_i$ in the magma at the ice–rock interface is equal to $P_t$ at the moment sill injection begins and increases thereafter, but must always be less (because of the pressure difference required to drive magma motion against wall friction losses) than the pressure in a static column of magma extending from the reservoir up to this point, which Figure 1 shows to be $P_c$ given by

$$P_c = P_a + g y \rho_i + g z \rho_t - g z \beta$$

$$= P_a + g y \rho_i + g z (\rho_t - \beta)$$

(8)

where $P_a$ is the atmospheric pressure, c. 0.1 MPa. Earlier we specified that $y$ would probably lie in the range 500 to 2000 m and that $z$ would lie in the range 1000 to 3000 m. Also, we found that the bulk magma density averaged over the vertical extent of the feeder dyke would lie within c. 10% of $2250 \text{ kg m}^{-3}$. Thus if $\rho_t$ equals $2300 \text{ kg m}^{-3}$, the value of $(\rho_t - \beta)$ will lie between about +75 and $-175 \text{ kg m}^{-3}$. Then since $\rho_i$ equals $917 \text{ kg m}^{-3}$, $P_c$ is dominated by the first term in equation (8) and is only a little greater than the weight of the overlying lithostatic (cryostatic) load. Thus as the sill grows, $P_i$ increases from $P_t$ towards $P_c$, just attaining this value when sill growth ceases and the pressure gradients due to magma motion vanish. Furthermore, Rubin (1993) showed that most of the pressure decrease used to overcome friction will occur over a disproportionately short distance near the dyke tip. As a result, the pressure in nearly all of the sill will be quite close to $P_c$ for most of the duration of its emplacement after the brief initial period when most of the sill consists of ‘tip’.

We use this fact in Table 1 to illustrate conditions in a typical mafic magma intruded at the interface between glaciers of various thicknesses, $y$ and the underlying silicate surface. The magma reservoir depth is assumed to be just greater than 1 km so that $[g z (\rho_t - \beta)] = 0.5 \text{ MPa}$, and so the sill pressure $P_c$ exceeds the overlying load by this amount. At great depths the magma contains 0.25 mass% H$_2$O and 0.2 mass% CO$_2$ as before; the table shows the amounts of these volatile phases exsolved, the bulk magma density, and the volume proportion of the magma consisting of gas bubbles for a sill intruded under various ice thicknesses ($y$) from 50 m to 2000 m. The entry in Table 1 for $y = 303$ m corresponds to a sill pressure of 3.3 MPa, which is the water saturation pressure for the assumed water content of 0.25 mass%. The fact that under shallower ice thicknesses the sill inlet pressure must be less than this value inevitably implies that excessive amounts of water vapour would have to be exsolved in the dyke tip during such intrusions. Indeed, it calls into question the advisability of ever assuming that the pressure in
Table 1. Illustration of the amounts of CO₂ and H₂O exsolved from a mafic magma intruding beneath glacial ice layers of various thicknesses and the consequences for the bulk density of the magma and the volume fraction of the magma that consists of gas bubbles

<table>
<thead>
<tr>
<th>Glacial ice thickness (m)</th>
<th>Pressure in most of sill (MPa)</th>
<th>Exsolved CO₂ amount (mass%)</th>
<th>Exsolved H₂O amount (mass%)</th>
<th>Magma bulk density (kg/m³)</th>
<th>Exsolved gas proportion (volume%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.049</td>
<td>0.19903</td>
<td>0.13856</td>
<td>557</td>
<td>79.4</td>
</tr>
<tr>
<td>100</td>
<td>1.499</td>
<td>0.19876</td>
<td>0.10692</td>
<td>817</td>
<td>69.8</td>
</tr>
<tr>
<td>250</td>
<td>2.847</td>
<td>0.19795</td>
<td>0.02583</td>
<td>1600</td>
<td>40.9</td>
</tr>
<tr>
<td>303</td>
<td>3.327</td>
<td>0.19766</td>
<td>0.0</td>
<td>1869</td>
<td>30.9</td>
</tr>
<tr>
<td>500</td>
<td>5.093</td>
<td>0.19660</td>
<td>0.0</td>
<td>2096</td>
<td>22.5</td>
</tr>
<tr>
<td>1000</td>
<td>9.587</td>
<td>0.19391</td>
<td>0.0</td>
<td>2348</td>
<td>13.2</td>
</tr>
<tr>
<td>1500</td>
<td>14.080</td>
<td>0.19121</td>
<td>0.0</td>
<td>2454</td>
<td>9.3</td>
</tr>
<tr>
<td>2000</td>
<td>18.573</td>
<td>0.18852</td>
<td>0.0</td>
<td>2513</td>
<td>7.1</td>
</tr>
</tbody>
</table>

The total volatile content of the magma prior to any gas exsolution is 0.2 mass% CO₂ and 0.25 mass% H₂O. See text for discussion.

Table 2. Examples of the minimum ice thicknesses needed to suppress spontaneous magma disruption during sill injection as a function of the magma water content

<table>
<thead>
<tr>
<th>Magma water content (mass%)</th>
<th>Water saturation pressure (MPa)</th>
<th>Required minimum ice thickness (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>24.0</td>
<td>563</td>
</tr>
<tr>
<td>0.5</td>
<td>9.0</td>
<td>245</td>
</tr>
<tr>
<td>0.25</td>
<td>3.3</td>
<td>98</td>
</tr>
<tr>
<td>0.125</td>
<td>1.2</td>
<td>28</td>
</tr>
</tbody>
</table>

The magma is assumed to contain 0.2 mass% CO₂ in addition to the amount of H₂O shown.
buoyancy (given by \[\frac{4}{3}\pi r^3(\beta - \sigma_g)g\] where \(r\) is the bubble radius, \(\beta\) is the magma density, \(\sigma_g\) is the gas density and \(g\) is the acceleration due to gravity) and the drag force acting on them (\(6\pi r\eta_m u\) where \(\eta_m\) is the magma viscosity). Using \(\beta = 2250\ \text{kg m}^{-3}, g = 9.8\ \text{m s}^{-2}\) and \(\eta_m = 100\ \text{Pa s}\) for a mafic magma, and neglecting \(\sigma_g\) because it is approximately 100 times smaller than \(\beta\), the rise speed of a 93 micron diameter carbon dioxide bubble is \(0.42\ \mu\text{m s}^{-1}\). At this speed, about four weeks are needed to segregate all of the bubbles from a sill one metre thick; proportionally greater times are needed for thicker sills, and sills intruded under thicker ice layers will have smaller bubbles with longer rise times. These timescales are much longer than the emplacement times of the sills (at most a few hours given the typical rise speeds of mafic magmas) and so gas loss can be ignored in all cases.

Magma intruding into a sill spreads sideways and, if the ice–rock interface is inclined, preferentially downslope. Magma in a sill probably always forms a thinner and more widespread layer than lava in a surface flow with the same mass flux and hence causes a more geometrically efficient transfer of heat to the ice; this may be a second explanation for the initial abundance of meltwater that is observed in some Icelandic subglacial eruptions (Gudmundsson et al. 1997). We base this assertion on the following series of arguments. The thicknesses of subaerial flows are determined by the bulk density, viscosity and effective yield strength of the magma, and the effect due to gravity, and the surface slope: the requirements are that in the levees the stress at the base (the product of levee thickness, gravity and ground slope) must equal the volume flux from the vent (e.g. Pinkerton & Wilson 1994). The same is true for pahoehoe toes with the complication that a yield-strength-like component of the magma rheology, in addition to the other factors, influences the 'central channel' depth. In contrast to this, a subglacial flow or sill has no free upper surface. The thicknesses of the 'levees' and the 'central channel' are controlled only by the stress distribution in the host rocks. On the largest spatial scales, that same stress distribution prevents the sill from thickening locally into a series of lava flow-like fingers in the same way that vertically propagating dykes travel upward as sheets of finite lateral extent, not as a series of nearby tubes. On smaller spatial scales, especially in the early stages of growth of a sill while it is still thin, there may be the possibility of minor instabilities causing the front of the sill to grow initially as a series of pahoehoe-like toes; as the sill extends and thickens, however, we expect any such toes to be overridden by the more nearly sheet-like intrusion.

To quantify some of these considerations, consider a basaltic shield volcano having a magma reservoir within which an excess pressure of 1 MPa causes a dyke to propagate to the surface. The length of the dyke, \(A\), is equal to the depth of the roof of the reservoir, say 2 km (e.g. Gudmundsson 1987; Ryan 1987). The mean width of the dyke, \(W\), will be given by

\[W = \frac{[(1 - \nu)\mu/(\pi/2)PA]}{\eta_m A}, \quad (9)\]

where \(\nu\) and \(\mu\) are the Poisson’s ratio and shear modulus for the crustal rocks, c. 0.25 and 3 GPa, respectively (Rubin 1993), so that \(W\) equals 0.8 m. The excess pressure drives magma with viscosity \(\eta_m\) upward through the dyke at an average speed \(U_M\) where, if the magma motion is laminar,

\[U_M = \frac{(W^2P)/(8\eta_m A)}{\eta_m A}. \quad (10)\]

If \(\eta_m = 100\ \text{Pa s}, U_M = c. 0.4\ \text{m s}^{-1}\). The Reynolds number for the magma motion is

\[Re = \frac{(2WU_M\beta)/\eta_m}{\eta_m}. \quad (11)\]

where \(\beta\) is the magma density, \(c. 2200\ \text{kg m}^{-3}\), in which case \(Re = c. 14\), confirming that the magma motion is laminar. The total volume flux, \(V\) through the dyke is the product of the magma speed \(U_M\), the dyke width \(W\) and the horizontal extent of the dyke, \(L\). Assuming that \(L\) is of the same order as \(A\), say 1 km, we find \(V\) equals \(320\ \text{m}^3\ \text{s}^{-1}\). For comparison, this value is quite similar to the \(c. 200\ \text{m}^3\ \text{s}^{-1}\) eruption rates typical of recent basaltic activity on the East Rift Zone of Kilauea volcano, Hawai‘i (Wolfe et al. 1987; Parfitt & Wilson 1994).

Assume first that this dyke feeds a subaerial basaltic lava flow which has a thickness \(D\), density \(\rho\), effective yield strength \(Y\), viscosity \(\eta_L\), and is moving down a slope \(\alpha\). Heslop et al. (1989) analysed the fluid mechanics of the proximal parts of a flow on the south edge of the summit caldera of Kilauea volcano for which typically \(D\) is \(c. 2\ \text{m}, \rho\) is \(c. 1000\ \text{kg m}^{-3}\), \(Y\) is \(c. 700\ \text{Pa}, \eta_L\) is \(c. 50\ \text{Pa s}\) and \(\alpha\) is \(2^\circ\). The mean advance speed, \(U_L\), of such a flow is given by

\[U_L = \frac{(\rho g \sin \alpha D^2)/(3\eta_L)}{\eta_L}. \quad (12)\]

so that in this case \(U_L\) is \(c. 9\ \text{m s}^{-1}\). To accommodate the total estimated magma volume flux of \(V = 320\ \text{m}^3\ \text{s}^{-1}\), the width of the flow must
Then be about 18 m, in good agreement with the observed width.

Now assume that, instead of erupting, the dyke magma ceases to propagate upward when it encounters the base of an ice layer 500 m thick (so that the magma density is close to 2100 kg m\(^{-3}\), see Table 1) and intrudes as a horizontal sill. Initially the sill will extend along the entire 1 km horizontal extent of the dyke (\(L\)) and will be growing laterally away from it on both sides (Figs 4 & 5). Let the proximal sill thickness be \(d_s\) (Fig. 5) and the magma flow speed be \(U_s\). The total volume flux must be the same as that in the dyke and so \(d_s\) and \(U_s\) are related via

\[
V = 2d_s L U_s. \tag{13}
\]

However, the sill grows by deforming the host materials (rock below, ice above) in an elastic manner, and the elastic properties of ice are not grossly different from those of rock (Hobbs 1974). Let the horizontal extent of the sill on either side of the dyke be \(E\) at time \(t\) and let the magma pressure at the point of injection be \(P_s\). Then by analogy with equation (9),

\[
d_s = \frac{[(1 - \nu)/\mu] \pi/2 P_s E}{(1 - \nu)\pi/2} \tag{14}
\]

By eliminating \(d_s\) between equations (13) and (14) and noting that by definition \(U_s\) equals \((dE/dt)\), we find the relationship \(V\) equals \(2[(1 - \nu)/\mu] \times \pi/2 P_s E L(dE/dt)\), which integrates to give \(E\) as a function of time:

\[
E = V/\left[(1 - \nu)/\mu \pi/2 P_s L\right]^{1/2} t^{1/2}, \tag{15}
\]

from which \(d_s\) can be found as a function of time by substituting equation (15) for \(E\) in equation (14):

\[
d_s = (1 - \nu)/\mu \left[\left(1 - \nu)/\mu\pi/2 P_s\right)^{1/2} V/L t^{1/2} t^{1/2}. \tag{16}\]

Finally, from \(U_s = (dE/dt)\) we have

\[
U_s = 0.5 V/\left[(1 - \nu)/\mu \pi/2 P_s L\right]^{1/2} t^{-1/2}. \tag{17}\]

Still using \(V\) equal to 320 m\(^3\) s\(^{-1}\) and \(L\) equal to 1 km, and taking \(P_s\) as 5 MPa, a suitable value for a sill intruded under about 500 m of ice (see Table 1), we find the values of \(E, d_s\) and \(U_s\) as a function of time shown in Table 3. After the first second the sill has grown horizontally to 13 m on

---

**Table 3.** Variation with time, \(t\) of the extent, \(E\), thickness, \(d_s\), and magma inflow speed, \(U_s\), for a sill driven by an injection pressure of \(P_s = 5\) MPa from a c. 0.8 m thick dyke 1 km in horizontal extent when the volume flux (\(V\)) is 320 m\(^3\) s\(^{-1}\)

<table>
<thead>
<tr>
<th>(t) (s)</th>
<th>(E) (m)</th>
<th>(d_s) (m)</th>
<th>(U_s) (m s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13</td>
<td>0.025</td>
<td>6.38</td>
</tr>
<tr>
<td>3</td>
<td>22</td>
<td>0.043</td>
<td>3.69</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
<td>0.079</td>
<td>2.02</td>
</tr>
<tr>
<td>30</td>
<td>70</td>
<td>0.14</td>
<td>1.17</td>
</tr>
<tr>
<td>100</td>
<td>128</td>
<td>0.25</td>
<td>0.64</td>
</tr>
<tr>
<td>300</td>
<td>221</td>
<td>0.43</td>
<td>0.37</td>
</tr>
<tr>
<td>1000</td>
<td>404</td>
<td>0.79</td>
<td>0.20</td>
</tr>
<tr>
<td>3000</td>
<td>699</td>
<td>1.37</td>
<td>0.12</td>
</tr>
<tr>
<td>10000</td>
<td>1277</td>
<td>2.51</td>
<td>0.064</td>
</tr>
<tr>
<td>30000</td>
<td>2211</td>
<td>4.34</td>
<td>0.037</td>
</tr>
</tbody>
</table>
either side of the dyke; its advance speed of 6 m s\(^{-1}\) is already smaller than the 9 m s\(^{-1}\) advance rate of the surface flow described above. Although the sill is only 25 mm thick, its surface area in contact with overlying ice by this time is 26,000 m\(^2\), whereas the surface area of the 2 m thick, 18 m wide lava flow after it has advanced 13 m is only 234 m\(^2\). Only if the sill grows to a distance of 6.4 km from the dyke will it have a mean thickness as large as the 2 m thickness of the lava flow; the magma injection speed of the sill will then be 0.1 m s\(^{-1}\), two orders of magnitude less than the lava flow advance rate.

Note that, even as early as one second after the start of sill injection, the thickness of the chilled skin on the sill magma, \(c.(\kappa_m t)^{1/2}\) equals 1 mm, where \(\kappa_m\) is the thermal diffusivity of basalt, is much less than the 25 mm thickness of the sill, so heat transfer to the overlying ice does not hinder sill injection in this example.

Admittedly, the above comparison has various deficiencies. For example, by the time the sill extends horizontally for a distance comparable to the along-strike length of the dyke (1 km in the earlier examples), it will be spreading sideways, i.e. its horizontal growth will be taking place parallel to the strike of the dyke as well as normal to it, and by the time it has extended to approximately twice this distance magma will be flowing more nearly radially away from the source region, and so the continuity relationship used above will overestimate the sill thickness and advance rate. Also, it has been tacitly assumed that the excess pressure in the magma is preserved throughout the emplacement event, whereas in fact that pressure is likely to decrease steadily as the magma reservoir at depth is deflated by magma removal. Further, the stress distribution in the feeder dyke has been assumed to remain constant, whereas in fact the growth of the sill will have a feedback effect on the overall geometry of the dyke–sill system, changing the magma volume flux to some extent. Even so, the comparison serves to support the assertion that sills generally have a larger contact area with adjacent ice than equivalent surface lava flows.

Some of the consequences of the injection of magma beneath an ice layer were investigated by Höskuldsson & Sparks (1997), who evaluated the variations with time of the thickness of the chilled crust on the magma and also the heat loss rate from the magma, and hence the thickness of ice melted. They did not, however, deal explicitly with the rate of thickening of the magma layer, instead introducing an efficiency factor which represented the fraction of the heat available from the magma that was actually transferred to the ice. Their analysis also effectively assumed that the overlying ice and underlying rock behave in a rigid fashion. The fact that water is denser than ice, leading to a volume decrease on melting, potentially provides some of the volume needed to accommodate the magma. Additionally, if the pressure in the water increases, some magma volume is accommodated by the small but finite amount of compression of the water produced (water is much more compressible than the overlying ice, the magma, or the underlying rock). If the water pressure becomes large enough to support the weight of the entire overlying ice layer, then sudden and large-scale (but short-lived) escape of the water along the margins of the ice–rock contact becomes possible.

We are not convinced that this is how the system behaves. The injection of magma into a sill fed by a dyke explicitly requires some deformation and local compression of the adjacent host materials as typified, for example, by the shear modulus and Poisson’s ratio in equation (9). The fact that the host material overlying the sill is ice rather than rock does not change this. Any water created by ice melting is a Newtonian fluid and transmits stresses isotropically (as does the unchilled part of the magma as long as its properties are near-Newtonian), so it is not inappropriate to consider pressure changes in the water independently of the pressure in the rest of the fluid system. Indeed, any potential pressure increase in the water (possibly caused, for example, by the very rapid conversion of a thin film of ice directly to supercritical vapour at the magma–ice contact) would first be accommodated by the compression of the bubbles of exsolved carbon dioxide in the adjacent magma.

In our view, the melting of ice into water during the intrusion process, and the consequent reduction in volume of the H\(_2\)O component (due to liquid water being denser than ice), simply makes it possible to inject a greater volume of magma for a given set of magma pressure conditions.

We do, however, agree with the analysis of Höskuldsson & Sparks (1997) as regards the rate of cooling of the injected magma and melting of the overlying ice, and now develop these ideas to illustrate the importance of the magma injection rate and the ultimate consequences of the intrusion process. Figure 5 shows schematically the thickening of the sill, its chilled crust and the overlying water layer, and defines the total thickness of the sill, \(d_e\), and the sill crust thickness, \(d_c\), near the sill injection point. The corresponding depth of ice melted is \(d_i\). Using treatments based on those developed by Carslaw & Jaeger (1947), Höskuldsson & Sparks (1997) give the crust thickness, \(d_c\), and the heat loss rate per unit
area of magma ice contact, \( q \), as a function of time, \( t \), as

\[
d_c = 2\lambda (\kappa_m t)^{1/2}, \tag{18}
\]

\[
q = \frac{[k_m (T_m - T_w)]}{[\text{erf}(\lambda) (\pi \kappa_m t)^{1/2}]}, \tag{19}
\]

where \( T_m \) is the temperature of the uncooled sill magma, \( T_w \) is the temperature of the meltwater above the crust, \( \kappa_m \) and \( k_m \) are the thermal diffusivity and thermal conductivity, respectively, of the solidified magma, and \( \lambda \) is a constant given by the solution of

\[
\frac{[\lambda^{-1} \exp(- \lambda^2)]}{[\text{erf}(\lambda)]]} = \left[ \frac{\pi^{1/2} L_m}{[\text{cm}(T_m - T_w)]} \right] \tag{20}
\]

where \( L_m \) and \( \text{cm} \) are the latent heat of fusion and the specific heat, respectively, of solidified magma. Taking \( L_m \) as \( 2.09 \times 10^5 \) J kg\(^{-1}\), \( \text{cm} \) as \( 1200 \) J kg\(^{-1}\)K\(^{-1}\), \( T_m \) as 1473 K (1200°C), and \( T_w \) as 277 K (i.e. close to the melting point and just above the temperature at which water has its maximum density) we find \( \lambda \) equals 1.1514 and \( \text{erf}(\lambda) \) equals 0.8968. We note that Hóskuldsson & Sparks (1997) found \( \text{erf}(k) = 0.84 \), and suspect that they inadvertently used the latent heat of fusion of ice, rather than that of magma, in solving equation (20), but this does not lead to any major differences between their results and ours.

We now integrate equation (19) to find the total amount of heat absorbed by the ice and the resulting water as a function of time, \( H(t) \):

\[
H(t) = \int_0^t q(t') dt' = \frac{[2k_m (T_m - T_w)]}{[\text{erf}(\lambda)]} \left[ t/(\pi \kappa_m t) \right]^{1/2} \tag{21}
\]

and equate this to the amount of heat needed to melt the thickness \( d_i \) of ice, \( d_i = H/(\rho_i L_i) \), where \( \rho_i \) and \( L_i \) are the density and latent heat of fusion of ice, respectively, giving

\[
d_i = \frac{[2k_m (T_m - T_w)]}{[\rho_i L_i \text{erf}(\lambda)]} \times \left[ t/(\pi \kappa_m t) \right]^{1/2} \tag{22}
\]

Using these results, the first five columns of Table 4 show how \( q, H, d_i \) and \( d_c \) are expected to change with time. Also shown is the thickness of the water layer produced by the ice melting, \( d_w \), given by \( d_w = d_i (\rho_i/\rho_w) = c.0.917 d_i \).

It has tacitly been assumed in the above analysis that the sill is injected fast enough that the total sill thickness at the vent, \( d_v \), is greater than the chilled crust thickness \( d_c \); in other words, there is some uncooled magma in the core of the sill. The calculations given above for the increase in the crust thickness with time therefore imply a minimum magma injection rate into the sill. In the example of sill injection calculated earlier for comparison with an equivalent volume-flux lava flow, we saw that the sill was easily able to avoid excess cooling. To establish the minimum magma volume flux to allow sill injection to be thermally viable, we note that the essential requirement is that the sill thickness \( d_s \) given by equation (16) must exceed the chilled crust thickness \( d_c \) given by equation (18). Both have the same time dependence, and so the requirement is simply

\[
\{(1 - v)/\mu [(\pi/2)P_s]^{1/2} \}^{1/2} (V/L)^{1/2} \gg 2\lambda \kappa_m^{1/2} \tag{23}
\]

which, since \( \lambda \) equals 1.1514, is more conveniently written

\[
\{(1 - v)/\mu [(\pi/2)P_s(V/L)]^{1/2} \gg 5.3 \kappa_m \tag{24}
\]

We saw in Table 1 that \( P_s \) probably lies between 3 and 18 MPa; \( \kappa_m \) is c. 0.8 \times 10^{-6} \) m\(^2\) s\(^{-1}\), and we have \( v = 0.25 \) and \( \mu = 3 \) GPa. Thus the requirement is essentially that \( (V/L) \) should be greater than a critical value which lies between \( 6 \times 10^{-4} \) and \( 36 \times 10^{-4} \) m\(^2\) s\(^{-1}\). Some values of \( (V/L) \) observed in, or deduced for, subaerial eruptions include c. 3 m\(^2\) s\(^{-1}\) for the 1961 fissure eruption at Askja, Iceland (Thorarinsson & Sigvaldason 1962), c. 0.6 m\(^2\) s\(^{-1}\) for the 1783 Lakagigar eruption in Iceland (Thorarinsson 1969), c. 7 m\(^2\) s\(^{-1}\) for the July, 1974 summit eruption of Kilauea, Hawai‘i (Heslop et al. 1989) and 12 m\(^2\) s\(^{-1}\) for the Yakima member of the Columbia River Basalt series (Swanson et al. 1975). These are all orders of magnitude greater than the minimum flux required, and so it seems likely that sill injection beneath ice should be a common occurrence, uninhibited by cooling problems, when the stress regime favours it.

Further stages of activity

When cooling does not limit sill injection at an ice–rock interface, magma injection will continue until one of two possible events happens: (1) the supply of magma from the source feeding the eruption ceases because the stresses driving the magma have been relaxed; (2) the sill spreads far enough laterally that the stresses at the propagating tip of the sill cause the precursor fracture (recall that the tip of the sill will contain pressurized water vapour and not magma) to reach the edge of the ice pile so that a connection is made to the atmosphere. We now consider the consequences of these events in turn.
Magma supply ceases

If the magma supply is cut off, growth of the sill immediately ceases, but melting of overlying ice continues as heat is convected upward through the water layer in contact with the sill. Because of the volume reduction as ice melts to water, the first consequence of this is that any residual excess pressure which may be present in the magma, together with any residual non-hydrostatic vertical compressive stress which may be present in the overlying ice, is quickly relaxed. Subsequently, if the overlying ice cannot deform downward fast enough, or alternatively if expansion of gas bubbles in the as-yet un-solidified part of the sill magma cannot crack the cooled crust and expand the sill sufficiently, a gap filled with water vapour will exist between the water and the overlying ice. Assuming that the temperature in the convecting water remains at only a few K above the melting point, the absolute pressure in the water would have decreased to \(c.10^3\) Pa, i.e. \(10^{-2}\) atmospheres, by the time that a vapour phase appeared. An absolute pressure this low would cause an enormous stress gradient in the overlying ice and an equally impressive pressure gradient across the chilled margin of the underlying sill, and so probably in practice no vapour layer ever forms. However, if it did so it would form a good insulator; the vapour density would be \(c.10^{-2}\) kg m\(^{-3}\) and so, although the specific heat of the vapour is only a factor of approximately two smaller than that of liquid water, the thermal capacity per unit volume of the vapour would be \(c.2 \times 10^2\) times smaller. Presumably in practice an equilibrium will be reached between ice deformation, sill inflation, ice melting and heat transfer in which an appropriately narrow vapour space exists (if it exists at all).

This process will continue until the available sill magma heat content is exhausted. An upper limit on the thickness of ice which can be melted by a given thickness of magma can be found by assuming that heat transfer through water and water vapour continues to cause ice melting until all of the magma has cooled to 274 K, the temperature at which water has its maximum density, at which point convective heat transfer ceases. On this basis, and assuming no net lateral transfer of heat, each one metre thickness of sill magma could melt \((\rho_m/l_m + c_m(T_m - T_w))/\rho_w = 14.5\) m thickness of ice, to form a water layer \(14.5 \times (\rho_m/\rho_w) = 13.3\) metres deep. The \((14.5 - 13.3 = )\) 1.2 metres of space thus created by the time ice melting ceases more than accommodates the one metre thickness of magma intruded, and so although the initial intrusion of the sill must have caused some small net uplift of the ice mass, subsidence of the overlying ice ultimately occurs. There seems no reason why any activity should be seen at the surface other than slow ice subsidence, greatest over the vent, to form an ice cauldron (Fig. 6). Only if the accumulated water escapes, either by slow seepage or by sudden release in a jökulhlaup, will there be more complex short term topographical changes (Björnsson 1992). Of course, if water does not escape, it will eventually freeze again, and its expansion as this happens will induce new stresses in the ice layer. However, the freezing process will be so slow (the timescale for conductive heat loss from under 100 m of ice is about 300 years) that any required ice deformation will probably be by plastic creep.

Finally, we note that the heat sharing calculation just employed assumes that all of the heat lost from the sill causes ice melting. This may not be the case. The temperature of the water between the sill and the ice is by definition higher than the ice melting point, whereas the ice itself must have a temperature at or below the melting point. If the ice temperature is even infinitesimally below the melting point, some heat is conducted into the ice ahead of the melting front and is not available to supply latent heat to melt ice. However, this is not a large effect. Consider the \(d = 4.3\) m thick sill intruded on a timescale of \(3 \times 10^4\) s = c. 8 hours illustrated in Table 3. According to the above calculation this sill could generate a \(4.3 \times 13.3 = c. 57\) m deep water layer. The timescale for cooling the sill is \(c.[d^2/\kappa_m] = c. 1.8 \times 10^7\) s = c. 200 days. On this time scale a thermal wave would penetrate a comparable \(c.4\) m distance into the ice ahead of the melting front. Assume that the ice was as much as 10 K below the freezing point. Then the average amount of ice heating would be \(c.5\) K and the amount of heat leaked into the ice per unit area would be \(c.4\) m \(\times 5\) K \(\times 2100\) J kg\(^{-1}\) K\(^{-1}\) \(\times 917\) kg m\(^{-3}\) = \(3.9 \times 10^7\) J m\(^{-2}\). The amount of heat contained in the \(c.57\) m thick layer of water (heated to 4 K above the melting point) would be \(c.57\) m \(\times 4\) K \(\times 4200\) J kg\(^{-1}\) K\(^{-1}\) \(\times 1000\) kg m\(^{-3}\) = \(9.6 \times 10^8\) J m\(^{-2}\). This suggests that the heat transfer to the water is more than 95% efficient. In contrast, the heat-sharing calculation employed earlier shows that the efficiency of the process would have to be less than 83% before there was no net subsidence of the ice.

There is a potentially useful diagnostic consequence of activity in which the intruded sill is never exposed to atmospheric pressure. With ice overburdens of several hundred metres, basaltic magmas should typically exsolve most of their CO\(_2\) but little of their H\(_2\)O. Thus, as pointed out
Fig. 6. Successive events during and after the intrusion of a sill at the base of an ice layer when the sill does not reach the edge of the ice sheet. (a) Early stage of intrusion; (b) sill has grown in all directions, chilled crust and overlying water lens are both thicker; (c) sill growth has ceased due to termination of magma supply, chilled crust and water lens have both thickened, and some subsidence of the surface of the ice has begun because the ice-to-water volume decrease has more than compensated for the sill thickness; (d) all available heat has been extracted from sill and vertical extents of water lens and subsidence have reached their maximum values.

by Dixon et al. (2002), analysis of the residual CO₂ and H₂O contents of eruption products should help distinguish between magma that has been emplaced under an ice overburden and that which has been erupted subaerially.

A pathway to the edge of the ice forms

As soon as a growing sill (Fig. 7a) approaches close enough to the edge of the ice cover that a direct connection between the intruded materials and the atmosphere is made (Fig. 7b), the pressure in the sill tip will decrease to that of the atmosphere as the pressurized water vapour escapes. The elastic constraints on the aspect ratio of the sill will then decay very quickly as the water which has already been produced above the sill begins to leak out onto the surrounding surface. For a short time, the pressure acting at the magma–water interface will become equal to the hydrostatic weight of the overlying ice; we showed earlier that the pressure in the sill is always fairly close to this value, so no major change in the overall magma flow rate through and into the sill will occur at this stage.

However, as soon as a significant amount of water has drained from above the sill, the pressure in this region will start to decrease toward atmospheric pressure, because the water can be replaced by atmospheric air leaking in. Only if the overlying ice can deform on a short enough timescale to replace the water will ice automatically stay in close proximity to the top of the sill magma. For a set of conditions similar to that envisaged here, Höskuldsson & Sparks (1997) calculated an ice deformation rate of order 1 mm s⁻¹, so if the rate of thinning of the water layer exceeds this value, the pressure will inevitably start to decrease.

Any pressure reduction in the sill will lead to an increase in the pressure difference between the
Fig. 7. Successive events during and after the intrusion of a sill at the base of an ice layer when the sill extends as far as the edge of the ice sheet. (a) Early stage of intrusion; (b) sill has reached edge of ice sheet and some water leakage begins; (c) much of the water generated earlier has drained out from beneath the ice and reduced interface pressure has allowed additional magma vesiculation; (d) pressure has become low enough near the drainage point for sill magma fragmentation to begin, enhancing the heat transfer to overlying ice; (e) all of the sill has been disrupted and a lava fountain exists at the outlet of the feeder dyke, rapidly eroding the overlying ice and feeding a subglacial lava flow; (f) the equivalent of stage (e) when the overlying ice has collapsed, greatly increasing the efficiency of thermal contact between the lava flow and ice.

bottom and the top of the dyke, and hence an increase, albeit probably small, in the magma flow rate through the dyke system. It may also have dramatic consequences, because it will lead to gas exsolution from the sill magma beneath the chilled crust. In the initial stages, the magma will simply vesiculate: existing carbon dioxide bubbles will expand and new bubbles of both \( \text{CO}_2 \) and \( \text{H}_2\text{O} \) will form at a rate which causes the magma to stay in physical contact with the overlying ice (Fig. 7c). This will lead to continued water production, and the system will tend toward a new equilibrium in which the pressure in the water is greatest near the dyke and least at the edge of the ice sheet, the resulting pressure gradient driving the water toward the exit. However, this state of affairs cannot persist for long. It seems inevitable that, as water drainage becomes more efficient, and the flowing water itself begins to melt and erode overlying ice, the pressure at the sill–ice contact will decrease to approach the atmospheric pressure. The key issue is then whether or not the magma contains enough volatiles so that at atmospheric pressure the volume fraction of gas bubbles in the magma becomes so great that magma fragmentation
begins to occur. For the plausible volatile mixture used earlier (0.2 mass% CO$_2$, 0.25 mass% H$_2$O), magma fragmentation would begin at about 1.2 MPa, i.e. 12 bars, and so we assume that such fragmentation is common.

Since the lowest pressure in the system must always be at the distal end of the sill closest to the connection to the atmosphere, it is in this region that magma fragmentation will begin. As the pressure in the space above the chilled magma crust decreases, the crust will initially prevent any response from the underlying magma. However, due to the presence of cooling cracks in its outermost parts, the crust is unlikely to have great strength. Once the pressure difference across the crust exceeds this strength it will fail, and an expansion wave will propagate vertically downward into the sill. The speed of the wave will be some fraction of the speed of sound in the vesicular magma, at most c. 100 m s$^{-1}$ (Kieffer 1977; Wilson & Head 1981). Thus for a sill a few metres thick (Table 3) the timescale will be only a few hundredths of a second. Passage of the expansion wave will fragment the magma, and expansion of the released gas through a pressure difference equal to the effective crustal strength will accelerate disrupted magma clots to impact the overlying ice (Fig. 7d). As an illustration, formulae given by Wilson (1980) for transient explosions show that if the effective strength were 1 MPa, then under a 500 m thick ice layer where the sill pressure was c. 5 MPa (see Table 1), expansion of the c. 0.2 mass% of CO$_2$ from 5 MPa to c. 4 MPa would generate speeds in the hot vesiculated pyroclasts up to c. 30 m s$^{-1}$. This should result in locally enhanced ice melting and magma chilling, and might be enough to trigger a sustained violent fuel-coolant type of interaction (Wohletz & McQueen 1984; Zimanowski et al. 1991). The products of the explosive mixing would be directed toward the exit to the atmosphere, and the wave of pressure reduction, vesiculation and fragmentation would also propagate from the distal end of the sill toward the feeder dyke. In this case the propagation speed would be a balance between the speed of the wave front into the unaffected sill magma (again some fraction of the local speed of sound) and the speed at which water and fragmented magma could be expelled from the discharge region. The rate of escape will be influenced mainly by the lateral extent of the sill; we saw earlier that the rate at which the overlying ice can deform downwards is not likely to be more than a few mm s$^{-1}$. This explosive fragmentation process is, of course, an excellent candidate for the origin of sudden jökulhlaup production. It is not clear what fraction of the fragmented magma would be washed out with the escaping water and what fraction would be left behind to form a hyaloclastite deposit.

A major change occurs when the wave of magma disruption reaches the feeder dyke. During the fragmentation process magma is still flowing up the dyke and being injected into the sill. However, the fragmentation process greatly reduces the frictional energy losses associated with magma motion in the sill and so as soon as all of the sill magma has been fragmented, the flow rate up the dyke will inevitably increase somewhat. The pressure at the dyke outlet will now be very close to atmospheric, and so the system will behave just as it would have done if the eruption had started subaerially. A chain of lava fountains will form along the dyke and magma clots falling from the fountains will begin to form lava flows (Head & Wilson 1989). The lava fountains will impinge on the overlying ice, greatly increasing the ice melting rate above the dyke (Fig. 7e). The resulting cavity ‘drilled’ into the overlying ice will grow upward until the subaerial height of the lava fountain is reached (Head & Wilson 1987), after which heat will only be transferred to the ice by radiation from pyroclasts in the fountain. From this time onward a new balance between ice subsidence and melting will be established but, if the eruption continues for long enough, it is clear that the explosive activity may eventually emerge through the ice; interaction with the water being produced will cause the activity observed to be phreatomagmatic. This scenario would be complicated somewhat if the ice layer above the now fragmented sill residue underwent fracturing and collapse rather than slow plastic deformation (Fig. 7f). In this case the pressure acting at the exit from the dyke would still be very close to atmospheric as long as there was a reasonably high porosity and permeability in the collapsed ice block pile, but the interaction between the magma and the ice would be more vigorous because of the tendency of ice blocks to settle as their bases were melted.

There is a second possible consequence of efficient water drainage once a pressure pathway to the atmosphere is established, one which is particularly applicable to magmas that do not have a large volatile content. As soon as the elastic constraint on the shape of the magma–ice contact is removed, the cross-sectional shape of the magma body is free to evolve under more local forces; specifically, magma should begin to concentrate into one or more structures resembling subaerial lava flows (Fig. 8a, b). The change will happen because the energy losses due to
THERMAL CONSEQUENCES OF SUBGLACIAL ERUPTIONS

Fig. 8. The development of subglacial lava flow structures. (a) Sill reaches edge of ice sheet and elastic constraints are relaxed but no explosive fragmentation of sill magma occurs; (b) water escapes and flow regime evolves to resemble that of subaerial flows. (c) Alternative source of subglacial lava flows formed when sill is explosively fragmented, dyke exit experiences atmospheric pressure, and flows are generated from a lava fountain over the vent (see Fig. 7c, f).

The corresponding sill was initially c. 1 km wide (the same horizontal length as the feeder dyke) and increased in thickness as it grew, the mean thickness reaching several metres after about 10 hours. Thus the thickness is largely irrelevant and the sill perimeter is somewhere between 1 and 2 km. The uncertainty arises because the base is always in contact with a stationary rock surface but the top has a layer of water between it and the stationary ice, thus making the frictional slip conditions more complicated. The same issue would apply to a subglacial lava flow, because even if its top were in contact with the overlying ice, there would be a layer of low-viscosity water, however thin, at the interface. Again this hardly matters, however, because even the conservative sill perimeter of c. 1 km is vastly greater than the worst case (2 x 18 =) 36 m friction-generating perimeter of the flow. Changing the assumed slope down which the flow-like structure moves would change its cross-sectional shape somewhat (note the presence of sin α in equation (12)), but again not enough to change the fact that any small instability which causes the advance of the magma to become concentrated into one or more flow-like structures will be favoured. Water generated by heat transfer into the ice will tend to be channelled along the side(s) of the flow(s), and the system will only remain stable as long as the pressure in the water is maintained high enough to suppress magma vesiculation to the point of fragmentation. If this occurs, one or more discrete lava flows will emerge from beneath the ice (Fig. 8b). However if instead fragmentation occurs, then the factors already discussed relating to subglacial explosive activity come into play (Fig. 7e, f), and new lava flow lobes will grow away from the dyke (Fig. 8c).

Summary

(1) With appropriate modifications, the principles used to analyze subaerial eruptions and intrusions (both dyke- and sill-like) in silicate rocks can be applied to eruptions under, into and through ice sheets, as illustrated in Figures 9 and 10. The geometries of dyke and sill emplacement and subsequent behaviour (decompression, transition to phreatomagmatic behaviour, etc.) are very efficient at delivering heat to the surrounding ice and creating high volumes of meltwater early in the eruptions, perhaps accounting for the production of major initial pulses of meltwater sometimes observed in Icelandic eruptions (e.g. Björnsson 1992).

(2) Typical basaltic magma densities and volatile contents are such that dykes which
Fig. 9. Diagrammatic representation of subglacial and englacial intrusions. At (1) the dyke may become a sill at the bedrock–ice interface, and subsequent heating of the ice can lead to meltwater production or, if drainage occurs, an ice cavern and transition to a flow. In (2) the dyke propagates a significant distance into the overlying ice, which appears rheologically similar to the underlying silicates at these strain rates; if enough volatile exsolution occurs, propagation to the surface may occur and an eruption plume could be produced. Heating and ice melting at the dyke margin causes it to lose coherence and collapse to form a rubble pile. Such rubble piles could lie at the cores of hyaloclastite ridges.

Fig. 10. Diagrammatic representation of key phases of subglacial eruptions. At (1) dyke intrusion leads to sill formation at the bedrock–dyke interface; at (2) heating produces a meltwater lens. If meltwater is drained and ambient atmospheric pressure is reached, phreatomagmatic eruptions will occur, accompanied by subsidence and ice cauldron formation; At (3) collapse of the ice surface can lead to Hawaiian or Surtseyan eruptions, depending on the involvement of meltwater in the vent, at (4).
Table 4. Variation with time, $t$, of the heat flux, $q$, and the total amount of heat released so far, $H$, from a sill intruded under ice. Also given are the thicknesses of the cooled crust on the sill, $d_c$, the ice layer melted, $d_i$, and the layer of water produced, $d_w$.

<table>
<thead>
<tr>
<th>$t$ (s)</th>
<th>$q$ (kW m$^{-2}$)</th>
<th>$H$ (MJ m$^{-2}$)</th>
<th>$d_c$ (m)</th>
<th>$d_i$ (m)</th>
<th>$d_w$ (m)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>2395</td>
<td>4.8</td>
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<td>0.0156</td>
<td>0.0143</td>
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<tr>
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<td>1383</td>
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<td>0.0270</td>
<td>0.0248</td>
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<tr>
<td>10</td>
<td>757</td>
<td>15.1</td>
<td>0.0066</td>
<td>0.0493</td>
<td>0.0452</td>
</tr>
<tr>
<td>30</td>
<td>437</td>
<td>26.2</td>
<td>0.0115</td>
<td>0.0854</td>
<td>0.0783</td>
</tr>
<tr>
<td>100</td>
<td>240</td>
<td>47.9</td>
<td>0.0210</td>
<td>0.1559</td>
<td>0.1430</td>
</tr>
<tr>
<td>300</td>
<td>138</td>
<td>83.0</td>
<td>0.0363</td>
<td>0.2710</td>
<td>0.2477</td>
</tr>
<tr>
<td>1000</td>
<td>76</td>
<td>151.5</td>
<td>0.0663</td>
<td>0.4931</td>
<td>0.4522</td>
</tr>
<tr>
<td>3000</td>
<td>44</td>
<td>262.4</td>
<td>0.1149</td>
<td>0.8541</td>
<td>0.7832</td>
</tr>
<tr>
<td>10000</td>
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<td>1.4299</td>
</tr>
<tr>
<td>30000</td>
<td>14</td>
<td>830.0</td>
<td>0.3630</td>
<td>2.7100</td>
<td>2.4770</td>
</tr>
</tbody>
</table>

would have reached the surface and erupted subaerially can, if they reach the surface under an ice sheet, penetrate 20 to 30% of the way through the ice and stall as dyke-like intrusions (Figs 1–3). In most cases there would be no surface manifestation of these events other than possible minor subsidence, but in some cases gas venting, surface disturbance, and even minor phreatomagmatic activity might be observable. Subsequent ice melting will render these intrusions unstable and they will collapse to form characteristic fragmental deposits at the base of the ice.

(3) Sills can form at the bases of ice sheets (Fig. 4). The pressures in the magnas in these sills will typically be c. 0.5 MPa higher than the lithostatic pressure of the overlying ice (Table 1) and at low magma water contents exsolution of mainly CO$_2$ will cause the sills to have vesicularities typically ranging from 10% (up to 2 km ice cover) to 30% (a few hundred metres ice cover). Under shallower ice depths and with high magma water contents (Table 2), enough water exsolution may occur that spontaneous magma fragmentation takes place and sills may be intruded largely as hyaloclastite deposits. Such intrusions can reach lateral extents of c. 1 km and thicknesses of 1–2 metres in c. 1 hour (Table 3).

(4) Comparison of the typical rates of increase of thickness of subglacial sills (Table 3) with the rate of growth of chilled crust as they interact with overlying ice (Table 4, Fig. 5) shows that cooling will almost never inhibit their emplacement; intrusion will continue until either the magma supply ceases or the sill reaches the edge of the ice sheet.

(5) If magma supply ceases before the sill magma reaches the edge of the ice sheet, all of the available heat is extracted from the magma over a long time scale and subsidence of overlying ice occurs to form an ice cauldron (Fig. 6).

(6) If magma supply continues after the sill magma reaches the edge of the ice sheet, the release of confining pressure can have several consequences (Figs 7a–e & 8a–c). Rapid water release (jökulhlaup formation) can occur, exacerbated by the explosive decompression of sill magma and enhanced heating of the overlying ice (Fig. 7d). A subglacial lava fountain will form over the feeder dyke, locally greatly increasing the ice melting rate (Fig. 7e), and a new subglacial lava flow or group of flows (Figs 7e, f & 8c) will form, the ice-melting efficiency of which will be enhanced if overlying ice collapses into the cavity vacated by disruption of the initial sill (Fig. 7f). Alternatively, if explosive decompression of the sill does not occur, the shape of the subglacial sill may evolve into that of one or more lava flow-like structures (Fig. 8c).

(7) A wide array of volcanic landforms has been observed on Mars (Hodges & Moore 1994). Application of the principles developed here to Mars provides criteria to assess possible examples of intrusion and eruption below polar deposits, ice fields, and glaciers (Garvin et al. 2000; Ghatan & Head 2001; Head & Wilson 2002). Discussions in the field with Magnus Gudmundsson, Snorri Snorrasson, Elsa Vilmundardottir, Sveinn Jakobsson, J. Smellie and I. Skilling are gratefully acknowledged. Comments on the manuscript by J. Smellie and two anonymous reviewers helped us to clarify a number of issues. We thank A. Côté for help in drafting. This paper is based on an invited presentation given at the Volcano/Ice Interaction meeting in Reykjavik, Iceland, in August, 2000. We gratefully acknowledge financial support from NASA through the Planetary Geology and Geophysics Program and the Mars Data Analysis Program, and from PPARC through grant PPC/G/S/2000/00521.
Appendix

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>vertical extent of dyke (m)</td>
</tr>
<tr>
<td>$D$</td>
<td>thickness of subaerial lava flow (m)</td>
</tr>
<tr>
<td>$E$</td>
<td>horizontal extent of sill on either side of feeder dyke (m)</td>
</tr>
<tr>
<td>$H$</td>
<td>amount of heat released by magma per unit area of ice contact (J m$^{-2}$)</td>
</tr>
<tr>
<td>$J_c$</td>
<td>constant in CO$_2$ solubility law, equal to $3.4 \times 10^{-6}$ (dimensionless)</td>
</tr>
<tr>
<td>$K_c$</td>
<td>constant in CO$_2$ solubility law, equal to $6 \times 10^{-12}$ (Pa$^{-1}$)</td>
</tr>
<tr>
<td>$K_w$</td>
<td>constant in water solubility law, equal to $6.8 \times 10^{-8}$ (Pa$^{-1}$)</td>
</tr>
<tr>
<td>$L$</td>
<td>horizontal extent of dyke (m)</td>
</tr>
<tr>
<td>$L_m$</td>
<td>latent heat of fusion of magma, equal to $2.09 \times 10^5$ (J kg$^{-1}$)</td>
</tr>
<tr>
<td>$L_{m,n}$</td>
<td>latent heat of fusion of magma, equal to $2.09 \times 10^5$ (J kg$^{-1}$)</td>
</tr>
<tr>
<td>$L_{m,r}$</td>
<td>latent heat of fusion of magma, equal to $2.09 \times 10^5$ (J kg$^{-1}$)</td>
</tr>
<tr>
<td>$P$</td>
<td>ambient pressure (Pa)</td>
</tr>
<tr>
<td>$P_a$</td>
<td>atmospheric pressure, equal to $c.10^5$ (Pa)</td>
</tr>
<tr>
<td>$P_c$</td>
<td>pressure in static magma column extending from reservoir to ice–rock interface (Pa)</td>
</tr>
<tr>
<td>$P_e$</td>
<td>pressure in dyke tip in excess of local lithostatic load (Pa)</td>
</tr>
<tr>
<td>$P_i$</td>
<td>magma pressure at ice–rock interface (Pa)</td>
</tr>
<tr>
<td>$P_{pt}$</td>
<td>pressure in dyke tip while dyke is propagating (Pa)</td>
</tr>
<tr>
<td>$P_r$</td>
<td>pressure in magma at roof of magma reservoir (Pa)</td>
</tr>
<tr>
<td>$P_s$</td>
<td>magma pressure at sill inlet from feeder dyke (Pa)</td>
</tr>
<tr>
<td>$P_t$</td>
<td>residual pressure in dyke tip after it comes to rest (Pa)</td>
</tr>
<tr>
<td>$Q$</td>
<td>universal gas constant, equal to $8.314$ (kJ kmol$^{-1}$ K$^{-1}$)</td>
</tr>
<tr>
<td>$T_m$</td>
<td>magma temperature, equal to $1473$ (K)</td>
</tr>
<tr>
<td>$T_w$</td>
<td>temperature of meltwater above chilled sill crust, equal to $277$ (K)</td>
</tr>
<tr>
<td>$U_s$</td>
<td>speed of magma flowing into sill (m s$^{-1}$)</td>
</tr>
<tr>
<td>$V$</td>
<td>volume flux of magma flowing though dyke (m$^3$ s$^{-1}$)</td>
</tr>
<tr>
<td>$W$</td>
<td>mean width of dyke (m)</td>
</tr>
<tr>
<td>$Y$</td>
<td>yield strength of subaerial lava, equal to $700$ (Pa)</td>
</tr>
<tr>
<td>$a$</td>
<td>constant used in equation (5b), equal to $[\rho_0 QT_{m}(n_i-J_c)]$ (kg m$^{-2}$ s$^{-2}$ mol$^{-1}$)</td>
</tr>
<tr>
<td>$b$</td>
<td>constant used in equation (5b), equal to $[mc(1-n_i+J_c)-\rho_0 QT_{m}K_c]$ (kg mol$^{-1}$)</td>
</tr>
<tr>
<td>$c$</td>
<td>constant used in equation (5b), equal to $[mcK_c]$ (m$^2$ mol$^{-1}$)</td>
</tr>
<tr>
<td>$c_m$</td>
<td>specific heat of solidified magma, equal to $1200$ (J kg$^{-1}$ K$^{-1}$)</td>
</tr>
<tr>
<td>$d$</td>
<td>distance penetrated by thermal changes due to conduction (m)</td>
</tr>
<tr>
<td>$d_c$</td>
<td>thickness of chilled crust on sill (m)</td>
</tr>
<tr>
<td>$d_i$</td>
<td>thickness of ice melted adjacent to sill (m)</td>
</tr>
<tr>
<td>$d_s$</td>
<td>thickness of sill near feeder dyke (m)</td>
</tr>
<tr>
<td>$d_w$</td>
<td>thickness of water layer produced by ice melting (m)</td>
</tr>
<tr>
<td>$e$</td>
<td>constant used in equation (5b), equal to $[\rho_m/(2K_c)]$ (kg m$^{-2}$ s$^{-2}$)</td>
</tr>
<tr>
<td>$f$</td>
<td>constant used in equation (5b), equal to $[(\rho_0 b)/(2K_c h)]$ (kg m$^{-4}$ s$^{-2}$)</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration due to gravity, equal to $9.8$ (m s$^{-2}$)</td>
</tr>
<tr>
<td>$h$</td>
<td>constant used in equation (5b), equal to $[(b^2-4ac)^{1/2}]$ (kg mol$^{-1}$)</td>
</tr>
<tr>
<td>$k_m$</td>
<td>thermal conductivity of solidified magma, equal to $3.1$ (W m$^{-1}$ K$^{-1}$)</td>
</tr>
<tr>
<td>$m_c$</td>
<td>molecular weight of CO$_2$, equal to $43.99$ (kg mol$^{-1}$)</td>
</tr>
<tr>
<td>$n$</td>
<td>weight fraction of water dissolved in basalt (dimensionless)</td>
</tr>
<tr>
<td>$n_c$</td>
<td>solubility of CO$_2$ in basalt (dimensionless)</td>
</tr>
<tr>
<td>$n_e$</td>
<td>weight fraction of CO$_2$ exsolved from magma (dimensionless)</td>
</tr>
<tr>
<td>$n_t$</td>
<td>total CO$_2$ content of magma (dimensionless)</td>
</tr>
<tr>
<td>$n_w$</td>
<td>solubility of water in basalt (dimensionless)</td>
</tr>
<tr>
<td>$q$</td>
<td>heat loss rate per unit area of magma–ice contact (W m$^{-2}$)</td>
</tr>
<tr>
<td>$r$</td>
<td>radius of gas bubble (m)</td>
</tr>
<tr>
<td>$t$</td>
<td>time (s)</td>
</tr>
<tr>
<td>$u$</td>
<td>rise speed of gas bubbles through magma (m s$^{-1}$)</td>
</tr>
<tr>
<td>$x$</td>
<td>depth of upper dyke tip below ice surface (m)</td>
</tr>
<tr>
<td>$y$</td>
<td>thickness of surface ice layer (m)</td>
</tr>
<tr>
<td>$z$</td>
<td>depth of magma reservoir below rock–ice interface (m)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>slope of ground under subaerial lava flow (degrees)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>bulk density of magma (kg m$^{-3}$)</td>
</tr>
<tr>
<td>$\beta_m$</td>
<td>mean bulk density of magma in dyke (kg m$^{-3}$)</td>
</tr>
<tr>
<td>$\eta_m$</td>
<td>viscosity of magma in dyke, equal to $100$ (Pa s)</td>
</tr>
<tr>
<td>$\eta_L$</td>
<td>viscosity of magma in subaerial flow, equal to $50$ (Pa s)</td>
</tr>
<tr>
<td>$\kappa_i$</td>
<td>thermal diffusivity of ice, equal to $c.10^{-6}$ (m$^2$ s$^{-1}$)</td>
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<td>thermal diffusivity of solidified magma, equal to $c.10^{-6}$ (m$^2$ s$^{-1}$)</td>
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<tr>
<td>$\lambda$</td>
<td>constant in heat transfer equation, equal to $1.1514$ (dimensionless)</td>
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<td>$\mu$</td>
<td>shear modulus of crustal rocks, equal to $3 \times 10^9$ (Pa)</td>
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<tr>
<td>$\nu$</td>
<td>Poisson's ratio of crustal rocks, equal to $0.25$ (dimensionless)</td>
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<tr>
<td>$\rho$</td>
<td>density of subaerial lava flow, equal to $1000$ (kg m$^{-3}$)</td>
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<tr>
<td>$\rho_c$</td>
<td>density of CO$_2$ gas (kg m$^{-3}$)</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>density of ice, equal to $917$ (kg m$^{-3}$)</td>
</tr>
</tbody>
</table>
\[ \rho_m \] density of basaltic magmatic liquid, equal to 2700 (kg m\(^{-3}\))

\[ \rho_c \] density of crustal rocks, equal to 2300 (kg m\(^{-3}\))

\[ \sigma_g \] gas density in bubble (kg m\(^{-3}\))

**References**


